Jérôme & Parameterized Algorithms and Complexity

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JM 2021 : Scientific Tribute to Jérôme Monnot

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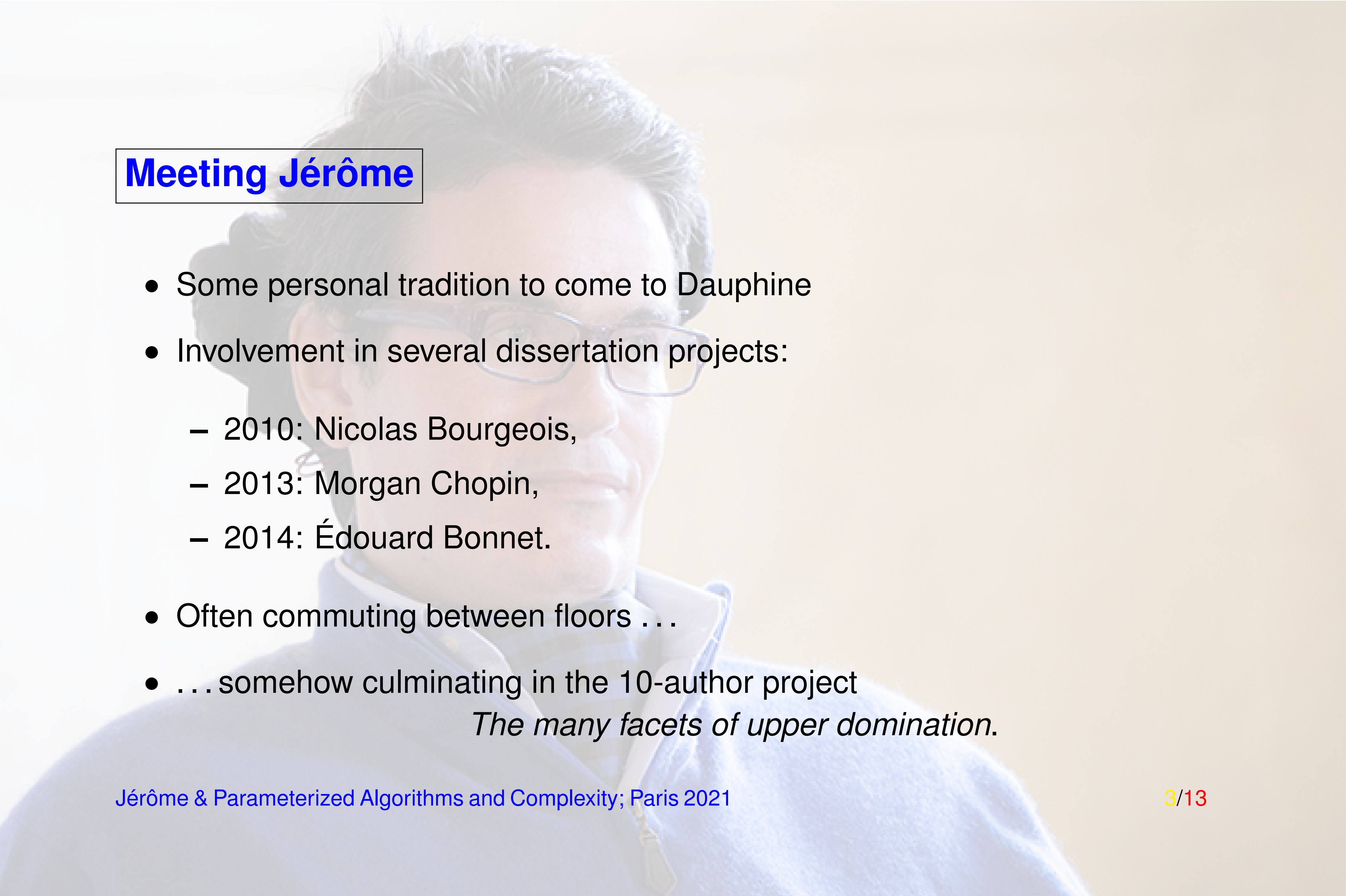
Overview

- How I met Jérôme
- The Upper Domination project
- What are extension problems?
  A framework for extension problems
- What about . . .
  (parameterized) complexity?
- Roman Domination
- Conference Program Design



Meeting Jérôme

- Some personal tradition to come to Dauphine
- Involvement in several dissertation projects:
  - 2010: Nicolas Bourgeois,
  - 2013: Morgan Chopin,
  - 2014: Édouard Bonnet.
- Often commuting between floors . . .
- . . . somehow culminating in the 10-author project
  The many facets of upper domination.

The UPPER DOMINATION Project

Given: a graph $G = (V, E)$
Task: Find an (inclusion-wise) minimal dominating set D of maximum size!

Our paper combined many results concerning approximation / parameterization of both groups. Examples of FPT- or W-results:

- With parameter pathwidth p: $\mathcal{O}^*(7^p)$. Open: $\mathcal{O}^*(c^p)$ for $c < 7$?
- With parameter treewidth t: $\mathcal{O}^*(10^t)$. Open: $\mathcal{O}^*(c^t)$ for $t < 10$?
- With lower-bound parameter k on D: W[1]-hard, in W[2].
  Open: Membership in W[1] or W[2]-hardness? Or anything in-between?
- With dual parameter $k_d = |V| - k$: Quadratic vertex & edge kernel, branching algorithm in $\mathcal{O}^*(4.3077^{k_d})$. Open: Improvements or lower bounds?

More on the Upper Domination Project

Given: a graph $G = (V, E)$
Task: Find an (inclusion-wise) minimal dominating set D of maximum size!

Open until today: Find exact algorithm for Upper Domination that is better than the one enumerating all minimal dominating sets!*

Our hope: Find methods how to cut search tree branches at an early stage.

We therefore introduced the following *extension problem*:
Given: a graph $G = (V, E)$ and $U \subseteq V$
Question: Is there a minimal dominating set containing U?

An efficient solution might help find a clever algorithm for Upper Domination.
Alas: The question is NP-hard in quite restricted scenarios.
Also: W[3]-complete when parameterized by $|U|$.

*$\mathcal{O}^*(1.7159^n)$ by Fomin, Grandoni, Pyatkin, Stepanov, ACM TALG 2008

Extension Framework (inspired by the def. of *NPO*); Example DS

A *monotone problem* is described as $\Pi = (\mathcal{I}, presol, sol, \preceq, m)$ with

- $\mathcal{I}$ is the set of *instances*, recognizable in poly-time. all graphs $G = (V, E)$
- For any $I \in \mathcal{I}$, $presol(I)$ is the set of *pre-solutions*. 2^V
  Moreover, for any $y \in presol(I)$, $|y|$ is polynomially bounded in $|I|$. ✓
- For $I \in \mathcal{I}$, $sol(I) \subseteq presol(I)$ is the *set of solutions*. dominating sets D
- '$U \in presol(I)$?' and '$U \in sol(I)$?' are decidable in poly-time on (I, U). ✓
- For $I \in \mathcal{I}$, $\preceq$ is a poly-time decidable partial ordering on $presol(I)$. inclusion $\subseteq$
- For $I \in \mathcal{I}$, $sol(I)$ is upward closed with respect to $\preceq$. ✓
- For $I \in \mathcal{I}$ & $U \in presol(I)$, $m(I, U) \in \mathbb{Q}_{\geq 0}$ is the poly-time computable *value of U*.
  cardinality $|D|$
- For $I \in \mathcal{I}$, $m(I, \cdot)$ is *monotone* with respect to $\preceq$, i.e., for all $U, U' \in presol(I)$ with $U' \preceq U$,
  - either $m(I, U') \leq m(I, U)$, so that $m(I, \cdot)$ is *increasing*, ✓
  - or $m(I, U') \geq m(I, U)$, so that $m(I, \cdot)$ is *decreasing*.

Extension Problems

Let $\Pi = (\mathcal{I}, presol, sol, \preceq, m)$ be a monotone problem.
$\mu(sol(I))$ denotes the set of *minimal feasible solutions of I*, i.e.,

$$\mu(sol(I)) = \{S \in sol(I) : ((S' \preceq S) \wedge (S' \in sol(I))) \to S' = S\} .$$

On $U \in presol(I)$, define $ext(I, U) = \{U' \in \mu(sol(I)) : U \preceq U'\}$: the set of *extensions* of U.

Sometimes, $ext(I, U) = \emptyset \rightsquigarrow$ the next question is interesting.

Ext Π
Input: $I \in \mathcal{I}$ and some $U \in presol(I)$.
Question: $ext(I, U) \neq \emptyset$?

Are there supersets of a given vertex set U that are inclusion-wise minimal dominating sets?

Motivation: Having arrived at pre-solution U with $ext(I, U) = \emptyset$: Stop branching!

A General Upper Bound on Complexity

If Π is a monotone problem, then EXT Π can always be solved within Σ_2^p.

Recall: $NP \cup \text{co-}NP \subseteq \Sigma_2^p$.

Given an instance (I, U) of EXT Π, we can perform the following steps.

1. Guess a solution U' of I. $\exists U' \in sol(I)$
2. Verify that $U \preceq U'$ holds, i.e., that U' is an extension of U.
3. Set the Boolean variable b to `false`.
4. For all solutions U'' of I do: $\forall U'' \in sol(I)$
   - Let $b := (U'' \preceq U') \wedge (U'' \neq U')$.
   - If b, then U' is not a minimal extension; exit the for-loop.
   - If not b, continue with the for-loop.
5. If (and only if) not b, then U' is a minimal extension.

Notice: Polynomial bound on solution size needed, but not upward closedness.

Parameterized Complexity

Define the *standard parameter* for Ext Π to be $m(I, U)$ on instance (I, U). The *dual parameter* is $\kappa_d(I, U) = m_{max}(I) - m(I, U)$ with $m_{max}(I) = \max\{m_I(y) : y \in presol(I)\}$.
If $m_{max}(I)$ is defined for all $I \in \mathcal{I}$, then Π *admits a dual parameterization*.
Define $Above(U) = \{V \in sol(I) : U \preceq V\}$.

Let $\Pi = (\mathcal{I}, presol, sol, \preceq, m)$ be monotone(, admitting a dual parameterization).
If, for all $I \in \mathcal{I}$ and $U \in presol(I)$, $Above(U)$ can be enumerated in *FPT*-time, parameterized by $k \in \{m(I, U), \kappa_d(I, U)\}$, then Ext Π is in *FPT*, parameterized by k.

In order to enumerate $Above(U)$, it is often easiest to enumerate $\{V \in presol(I) : U \preceq V\}$ instead (in *FPT*-time) and check if the enumerated pre-solution is a solution, doable in poly-time.

Param. \ Ext. of	EC	EM	EDS	IS	VC	DS	BP
standard	*FPT*	*FPT*	$W[1]$-hard	*FPT*	$W[1]$-compl.	$W[3]$-compl.	*para-NP*
dual	*FPT*	*FPT*	*FPT*	$W[1]$-compl.	*FPT*	*FPT*	*FPT*

Further Orderings but subset or superset . . .

Ask the Romans for help: **Roman Domination***. We only present the monotone problem Π_R. $f : V \to \{0, 1, 2\}$ is called a *Roman domination function* iff, for all vertices x with $f(x) = 0$, there is some $y \in N(x)$ with $f(y) = 2$.

$$\mathcal{I} = \{G = (V, E) : G \text{ is a graph}\}$$

$presol(G) = \{0, 1, 2\}^V$, polyn. bounded ✓, poly-time decidable ✓

$sol(G) = \{f \in presol(G) : f \text{ is a Roman domination function of } G\}$,
poly-time decidable ✓

$\preceq = \leq$, lifted 'point-wise', poly-time decidable ✓, $sol(G)$ upward closed ✓

$m(I, g) = g(V) = \sum_{x \in V} g(x)$, poly-time computable ✓

$$\mu(sol(G)) = \{f \in sol(G) : ((f' \preceq f) \wedge (f' \in sol(G))) \to f' = f\}$$

$$ext(G, f_U) = \{f \in \mu(sol(G)) : f_U \preceq f\}$$

Good news: Kevin Mann could prove: EXT ROMAN DOMINATION is poly-time solvable.
Alas, this does not help improve exact algorithms for ROMAN DOMINATION (see PhD of Liedloff).

*Stewart, *Scientific American* 1999

Open Parameterizations

Sometimes, open problems can be found in Jérôme's papers. WINE 2016

CONFERENCE PROGRAM DESIGN, or CPD for short:
Given: m talks $T = \{t_1, \ldots, t_m\}$ and n participants of a conference.
The conference should be run using k time slots.
Each slot contains at most q talks (held in parallel tracks).
Conference schedule: described by a partition $\mathcal{S} = \{S_1, \ldots, S_k\}$ with $|S_i| \leq q$.
Each participant is modeled by a utility function $u_\ell : T \to \mathbb{R}_{\geq 0}$.
Goal: maximize the overall utility, which is $\sum_{\ell=1}^{n} \sum_{i=1}^{k} \max\{u_\ell(t) \mid t \in S_i\}$.

If all preference orders $\prec_\ell$ induced by u_ℓ are single-peaked wrt. some linear order $\sqsupseteq$ on T, then Fotakis, Gourvès and Monnot showed an XP-algorithm wrt. parameter k for solving CPD.

Open question: Is there some FPT-algorithm for CPD? Or any lower bounds?

Thanks for your attention!

Theoretische Informatik Trier

International Workshop On Combinatorial Algorithms

See you soon at IWOCA