Jérôme Monnot & Graph Theory

Bernard Ries

Decision Support & Operations Research
Department of Informatics
University of Fribourg, Switzerland

JM2021: Scientific tribute to Jérôme Monnot

1. Introduction

2. Upper Domination in monogenic classes

A real challenge!

14H30 - 17H40 : SCIENTIFIC TALKS (A709)

- **14h30-14h55** : *Jérôme & Polynomial approximation and complexity* - Bruno Escoffier
- **14h55-15h20**: *Jérôme & Computational social choice* - Jérôme Lang
- **15h20-15h45**: *Jérôme & Algorithmic game theory* - Dimitris Fotakis

- **Cofee Break (A703)**

- **16h00-16h25** : *Jérôme & Parameterized algorithms and complexity* - *Henning Fernau (online)*
- **16h25-16h[illegible]**: *Jérôme & Multi-Objective optimization* - Fanny Pascual
- **16h50-15h15**: *Jérôme & Graph theory* - Bernard Ries
- **17h15-17h[illegible]**: *Jérôme & Operational research and Combinatorial optimization* - Dominique de Werra

Fruitful collaboration

- H. AbouEisha, S. Hussain, V. Lozin, J. Monnot, B. Ries, V. Zamaraev, **Upper Domination: towards a dichotomy through boundary properties**, *Algorithmica 80 (2018) 2799-2817*
- H. AbouEisha, S. Hussain, V. Lozin, J. Monnot, B. Ries, V. Zamaraev, **A boundary property for upper domination**, *Lecture Notes in Computer Science 9843 (2016) 229-240, International Workshop on Combinatorial Algorithms (IWOCA 2016)*
- V. Lozin, J. Monnot, B. Ries, **On the maximum independent set problem in subclasses of subcubic graphs**, *Journal of Discrete Algorithms 31 (2015) 104-112*
- M. Demange, J. Monnot, P. Pop, B. Ries, **On the complexity of the selective graph coloring problem in some special classes of graphs**, *Theoretical Computer Science 541 (2014) 89-102*

Fruitful collaboration II

- H. AbouEisha, S. Hussain, V. Lozin, J. Monnot, B. Ries, **A dichotomy for upper domination in monogenic classes**, *Lecture Notes in Computer Science 8881 (2014) 258-267, 10th Annual International Conference on Combinatorial Optimization and Applications (COCOA 2014)*
- N. Barrot, L. Gourves, J. Lang, J. Monnot, B. Ries, **Possible winners in approval voting** *Lecture Notes in Artifical Intelligence 8176 (2013) 57-70, Algorithmic Decision Theory (ADT 2013)*
- V. Lozin, J. Monnot, B. Ries, **On the maximum independent set problem in subclasses of subcubic graphs**, *Lecture Notes in Computer Science 8288 (2013) 314-326*, *International Workshop on Combinatorial Algorithms (IWOCA 2013)*
- M. Demange, J. Monnot, P. Pop, B. Ries, **Selective Graph Coloring on Some Special Classes of Graphs**, *Lecture Notes in Computer Science 7422 (2012) 320-331, International Symposium on Combinatorial Optimization (ISCO 2012)*

Selective graph coloring

- M. Demange, J. Monnot, P. Pop, B. Ries, **On the complexity of the selective graph coloring problem in some special classes of graphs**, *Theoretical Computer Science 541 (2014) 89-102*
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Upper dominating set problem

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Maximum independent set problem

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Approval voting

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Our approach: Exploiting graph structures to cope with hard problems

- We consider a problem Π for which we know/can show that it is NP-hard in general.

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- Or in other words, does the graph class $\mathcal{C}$ admit some "nice" property P which makes problem Π solvable in polynomial time in $\mathcal{C}$?
- P may consist in some structural property that we can exploit in order to come up with a polynomial-time algorithm for Π in $\mathcal{C}$.

Dominating Set

In a graph $G = (V, E)$, a *dominating set* is a subset of vertices $D \subseteq V$ such that any vertex outside of D has a neighbour in D. Such a set is said to be *minimal* if no proper subset of D is dominating.

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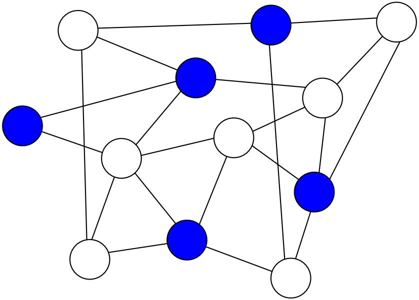

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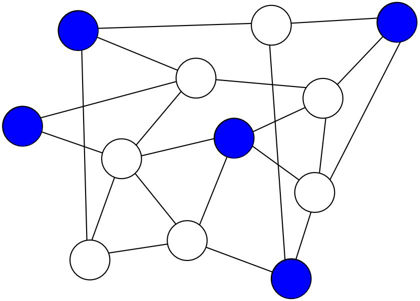

Complexity

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On the other hand, in some particular graph classes, the problem can be solved in polynomial time:

- bipartite graphs [Cockayne et al. 1981],
- chordal graphs [Jacobsen et al. 1990],
- generalized series-parallel graphs [Hare et al. 1987],
- graphs of bounded clique-width [Courcelle et al. 2000].

Complexity

Further results have been obtained regarding parametrised complexity and approximation:

- UPPER DOMINATING SET PROBLEM is $W[1]$-hard [Bazgan et al. 2016],
- for any $\epsilon > 0$, UPPER DOMINATING SET PROBLEM is not $n^{1-\epsilon}$-approximable, unless $P = NP$ [Bazgan et al. 2016].

Complexity

Further results have been obtained regarding parametrised complexity and approximation:

- Upper Dominating Set Problem is $W[1]$-hard [Bazgan et al. 2016],
- for any $\epsilon > 0$, Upper Dominating Set Problem is not $n^{1-\epsilon}$-approximable, unless $P = NP$ [Bazgan et al. 2016].

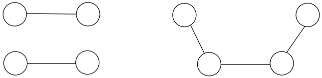
We were interested in the complexity of the Upper Dominating Set Problem in monogenic classes of graphs, i.e. classes defined by a single forbidden induced subgraph.

Let G and H be two graphs. Then G is said to be *H-free*, if it does not contain H as an induced subgraph.

Main result

Theorem [AbouEisha, Hussain, Lozin, Monnot, R., 2014]

Let H be a graph. If H is a $2K_2$ or P_4 (or any induced subgraph of $2K_2$ or P_4), then the UPPER DOMINATING SET PROBLEM can be solved for H-free graphs in polynomial time. Otherwise, the problem is NP-hard for H-free graphs



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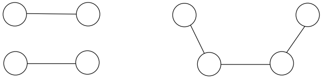

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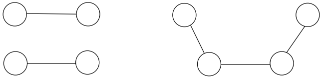

- In other words, we obtain a complete dichotomy in monogenic classes for the UPPER DOMINATING SET PROBLEM.
- Up to that date, a complete dichotomy in monogenic classes was available only for VERTEX COLORING [Král et al 2001], MINIMUM DOMINATING SET [Korobitsyn 1990] and MAXIMUM CUT [Kaminski 2012].

Hardness results

Theorem [AbouEisha, Hussain, Lozin, Monnot, R., 2014]

The UPPER DOMINATING SET PROBLEM is NP-hard when restricted to

a) planar graphs with maximum degree 6 and girth at least 6;
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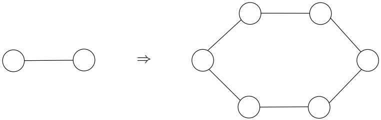
a) Reduction from Maximum Independent Set Problem in planar cubic graphs:



b) Reduction from Minimum Dominating Set Problem.

Positive results

Theorem [AbouEisha, Hussain, Lozin, Monnot, R., 2014]

The UPPER DOMINATING SET PROBLEM is polynomial-time solvable when restricted to

a) P_4-free graphs;

b) $2K_2$-free graphs.

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a) P_4-free graphs have cliquewidth at most 2 [Brandstädt et al., 2006].

b) Use structure of minimal dominating sets in $2K_2$-free graphs.

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- The class of $2K_2$-free graphs admits a polynomial-time algorithm to solve the MAXIMUM INDEPENDENT SET PROBLEM.
- Thus, we may restrict ourselves to the analysis of minimal dominating sets D such that
  - D contains at least one edge;
  - $|D| > \alpha(G)$.

$2K_2$-free graphs

Crucial structural result:

Theorem [AbouEisha, Hussain, Lozin, Monnot, R., 2014]

If a minimal dominating set in a $2K_2$-free graph G is larger than $\alpha(G)$, then it consists of a triangle and all the vertices not dominated by that triangle.

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This gives us the following algorithm:

1. Find a maximum independent set D in G.
2. For each triangle T in G
   - Let $D' = T \cup A(T)$.
   - If D' is a minimal dominating set and $|D'| > |D|$, then $D := D'$.
3. Return D.

Jérôme Monnot

- A humble and generous person!
- An excellent teacher and mentor/guide for young researchers!
- Very fruitful and rich collaboration!
- Interested in very different problems and areas!
- Significant contributions in many areas!

Thank you for your attention!