Jérôme, Algorithmic Game Theory and Strategyproof Facility Location

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Based on joint work with **Laurent Gourvès** (LAMSADE), **Jérôme Monnot** (LAMSADE) and **Panagiotis Patsilinakos** (NTUA)

Homage to Jérôme Monnot, Univ. Paris-Dauphine, December 2021

Jerómê and Algorithmic Game Theory

Spanning Trees Constructed by Selfish Agents [Gourvés M., WINE 08]

- Agents pay for **first edge** of path to root.
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- Cut games admit **strong** PNE and SPoA $= 2/3$.
- *k*-Cut games admit 3-strong PNE.
- If *k*-Cut games admit strong PNE (conjectured), SPoA $= \frac{2k-2}{2k-1}$.

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Congestion Games with Capacited Resources [Gourvés, M., Morreti, Thang, SAGT 12, ToCS 2015]

- PNE exists, $m = 2$ resources, NP-hard to tell for $m \geq 3$.
- PNE for **singleton games**, by elegant 2-dimensional potential!

Jerómê and Algorithmic Game Theory

Selfish Transportation [F. Gourvés M., SOFSEM 17]

- **Bus route** selection, based on flow time from entering bus until destination.
- PNE exist (and computed **efficiently**), if buses have "identical routes", or $m = 2$ and **metric** distances, or distances $\in \{1, 2\}$.
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- **Coordination mechanisms** for scheduling selfish tasks with setup times [Gourvés, M., Telelis, WINE 09]
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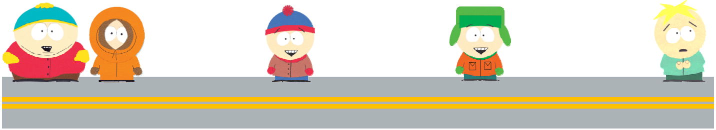
- **Project** games: how competitive agents select tasks and when decide to cooperate [Biló, Gourvés, M., CIAC 19]
- Fair allocation of indivisible goods: approximation of the **maximin share** for $n = 3$ and matroids [Gourvés, M., CIAC 17]

Our Focus: k-Facility Location Games

Public Good Allocation for Strategic Agents on the Line

- Agents $N = \{1, \ldots, n\}$ on the real **line** .
- Agent i **wants** a facility close to x_i , which is **private information** .



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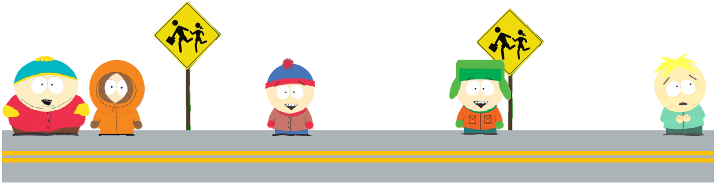
Mechanism F maps reported ideal locations $\boldsymbol{y} = (y_1, \ldots, y_n)$ to (probability distribution over) set(s) of k **facilities** .



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- Each agent i **reports** y_i that may be **different** from x_i.

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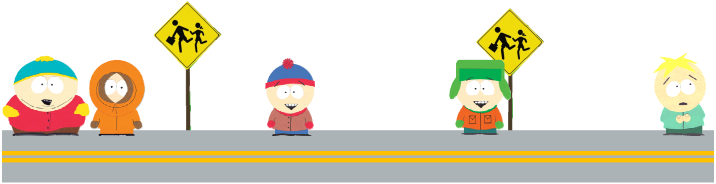
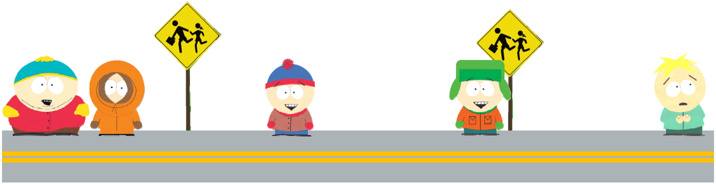


Preferences and Truthfulness

Connection Cost

(Expected) distance of agent i's **true location** to the **nearest** facility:

$$\text{cost}[x_i, F(\boldsymbol{y})] = \text{dist}(x_i, F(\boldsymbol{y})) = \min_{c \in F(\boldsymbol{y})} |x_i - c|$$



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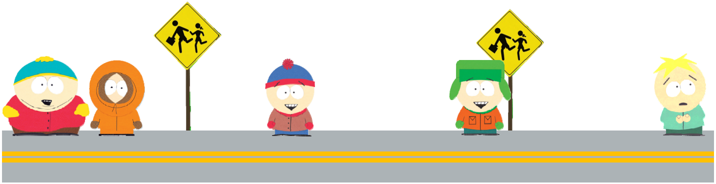
Truthfulness

For any location profile $\mathbf{x}$, agent i, and location y:

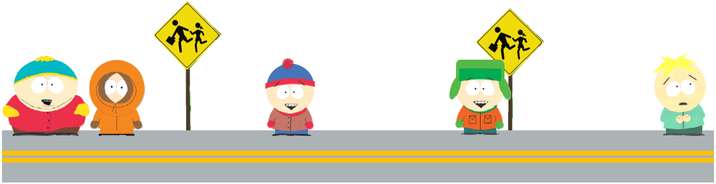
$$\text{cost}[x_i, F(\mathbf{x})] \leq \text{cost}[x_i, F(y, \mathbf{x}_{-i})]$$



Variants and Social Efficiency

Candidate Facility Locations:

- **Unrestricted** : Any point (esp. agent locations) can be facility.
- **Restricted** : Facilities selected from m candidate locations $\mathcal{C}$
  Motivation from multi-winner elections: Chamberlin-Courant.



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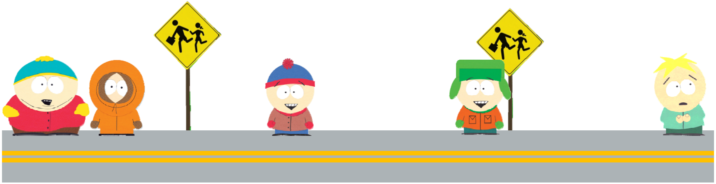
Social Objective

$F(\boldsymbol{x})$ should optimize (or approximate) a given **objective function** .

- **Social Cost** : minimize $\sum_{i=1}^{n} \mathrm{cost}[x_i, F(\boldsymbol{x})]$
- **Social Welfare** : maximize $\sum_{i=1}^{n} \left(L - \mathrm{cost}[x_i, F(\boldsymbol{x})]\right)$



1-Facility Location on the Line

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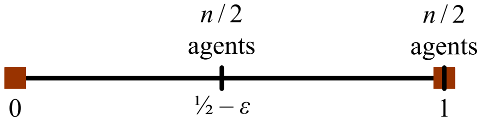
  - Median social cost $\approx 3n/4$. Median social welfare $\approx n/4$.



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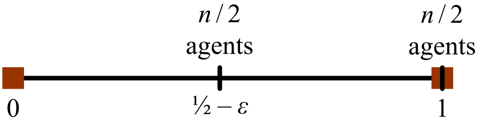
- Anonymity and truthfulness iff **generalized** median [Moulin 80]



k-Facility Location on the Line, $k \geq 2$

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Optimal is **not** truthful: optimal clustering **sensitive** to deviations!

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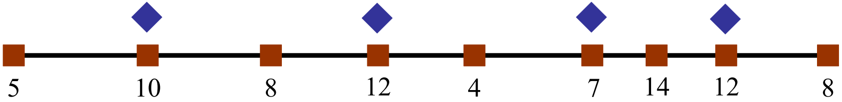
$(\alpha_1, \ldots, \alpha_k)$**-percentile** mechanism ($0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_k \leq 1$):

- vote(ℓ) = #agents preferring $\ell \in \mathcal{C}$ to other candidates in $\mathcal{C}$.
- j-th facility at leftmost $\ell \in \mathcal{C}$ with $\geq \alpha_j$ **fraction** of vote on ℓ **and its left**.
  - Median is 0.5-percentile. Two-Extremes is (0, 1)-percentile.



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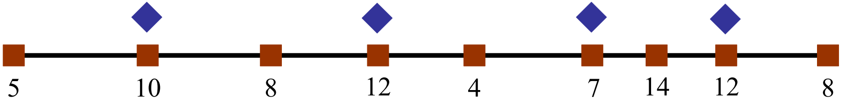
- Percentile mechanisms are **anonymous** and **truthful**.



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(k, q)-Conference Program Design
[Caragiannis Gourvés M., IJCAI 16], [F. Gourvés M., WINE 16]

Robust version of (kq)-Facility Location where each agent receives **utility from q different** facilities.

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- For any $q \geq 1$, $(1/(2k), 3/(2k), \ldots, (2k-1)/(2k))$-percentile applied to q-tuples is **truthful** and $(2k-3)/(2k-1)$**-approximate** .

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- Optimal solution through generalization of LP-based approach in [Hajiaghayi et al., SODA 14]

k-Facility Location – Social Cost

Truthful Location of 2 Facilities

Two-Extremes is $(n-2)$**-approximate** and best possible.
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- Focus on instances with **stable** optimal clustering.

Perturbation Stability for k-Facility Location

Perturbation Stability in Clustering [Bilu Linial, ITCS 10]

- **γ-stability** : scaling down any distances by factor $\leq \gamma$ (while maintaining metric property) **does not affect optimal** solution.

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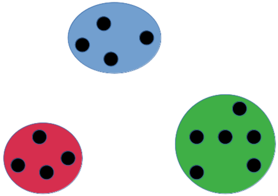
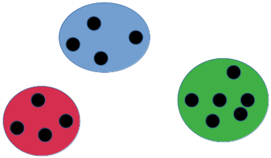
  k-Facility Location remains **hard** for $\gamma \leq 2 - \varepsilon$.





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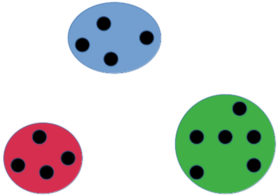
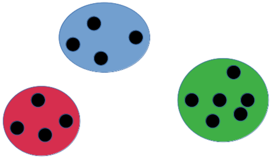
  k-Facility Location remains **hard** for $\gamma \leq 2 - \varepsilon$.
- Real-world instances are **stable**: *"Clustering is hard when it doesn't matter"* [Roughgarden 17]





Truthful k-Facility Location in Stable Instances

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Assume that **"true"** instances are indeed **stable** .

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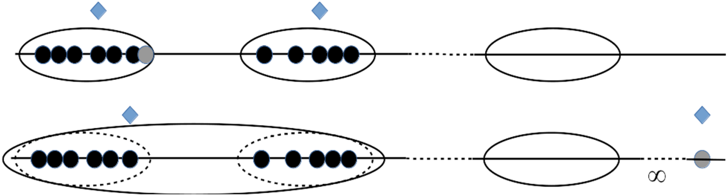

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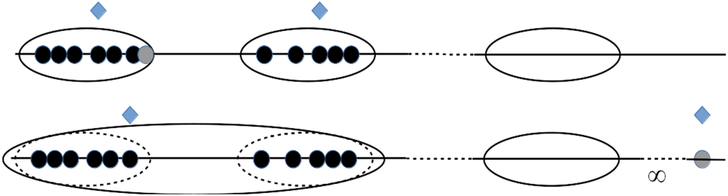
- Optimal solution **not truthful** for any stability $\gamma \geq 1$.
- For $k \geq 3$, deterministic anonymous truthful mechanisms for $(\sqrt{2} - \varepsilon)$**-stable** instances have **unbounded** approximation (based on [F. Tzamos, ICALP 13])



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Remedy and Main Results

- **Optimal** clustering $(C_1, \ldots, C_k)$ due to bounded approximation.
- Stability verification (necessary cond.): allocate facilities only if

$$\max\{\text{diam}(C_i), \text{diam}(C_{i+1})\} < d(C_i, C_{i+1})$$

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- For 5**-stable** instances, facility at **random** agent in each optimal cluster is **truthful** and 2**-approximate** .

Optimal Mechanism for Stable k-Facility Location

Optimal Mechanism and Approach to Truthfulness

If optimal clustering $(C_1, \dots, C_k)$ has **singleton** clusters or $\max\{\mathrm{diam}(C_i), \mathrm{diam}(C_{i+1})\} \geq d(C_i, C_{i+1})$, do **not allocate** facilities!

Otherwise, facilities at $(\mathrm{med}(C_1), \dots, \mathrm{med}(C_k))$.

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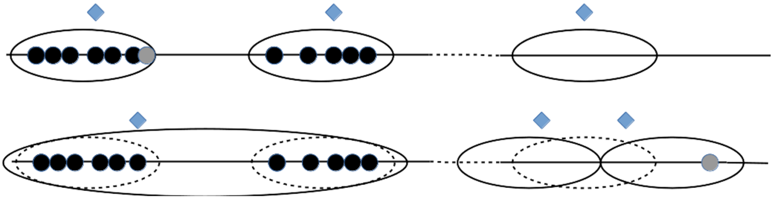
- Key deviation: rightmost agent of C_i deviates to C_j, causing C_j **to split** and C_i **to merge** with C_{i+1} .
- "Simulate" increase in cost of C_j by γ**-perturbation** and decrease in cost of C_j by agent's **cost improvement** .



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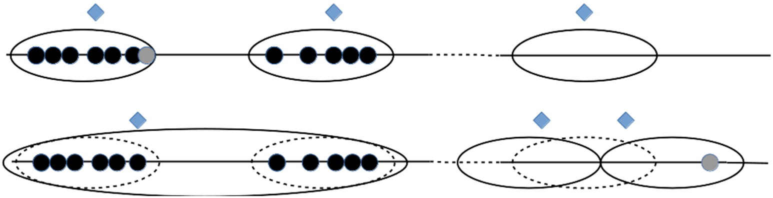
- "Simulate" increase in cost of C_j by **γ-perturbation** and decrease in cost of C_j by agent's **cost improvement** .
- Stability: optimal clustering **not affected** by deviation.



Research Directions

Open Questions

- Close the **gap in stability** for bounded approximation: lower bound of $\sqrt{2}$ and upper bound of $2 + \sqrt{3}$ (or 5).
- Extension to **trees** and study of **general metrics**.

Research Directions

Open Questions

- Close the **gap in stability** for bounded approximation : lower bound of $\sqrt{2}$ and upper bound of $2 + \sqrt{3}$ (or 5).
- Extension to **trees** and study of **general metrics** .
- Possibility of truthfulness **for all** instances and bounded approximation only **for stable** ? (conjecture: **no**)
- Complexity of **determining** whether a k-Facility Location instance is γ**-stable**, esp. for line and trees?

Thank You for Everything and Goodbye

We, your many friends, deeply miss your kindness,
openness, collaboration, passion for research and life,
warm smile and true love for people, and
so many things we kept learning from you.

Thank you and goodbye, Jérôme