# Jérôme and computational social choice

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1. voting under incomplete preferences (2010-2013 + 2016)
2. resource allocation and fairness (2012-2019)
3. multiwinner voting and proportional representation (2016-2019)

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- local : Nathanaël Barrot, Bernard Ries, Yann Chevaleyre, Bruno Escoffier, Laurent Gourvès, Jérôme Lang, Julien Lesca, Nicolas Maudet, Lydia Tlilane
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# Voting under incomplete preferences

- Yann Chevaleyre, Jérôme Lang, Nicolas Maudet, **JM** : Possible Winners when New Candidates Are Added : The Case of Scoring Rules. AAAI 2010
- Lirong Xia, Jérôme Lang, **JM** : Possible winners when new alternatives join : new results coming up ! AAMAS 2011.
- Yann Chevaleyre, Jérôme Lang, Nicolas Maudet, **JM** : Compilation and communication protocols for voting rules with a dynamic set of candidates. TARK 2011.
- Yann Chevaleyre, Jérôme Lang, Nicolas Maudet, **JM**, Lirong Xia : New candidates welcome ! Possible winners with respect to the addition of new candidates. Math. Soc. Sci. 2012
- Nathanaël Barrot, Laurent Gourvès, Jérôme Lang, **JM**, Bernard Ries : Possible Winners in Approval Voting. ADT 2013.
- Piotr Faliszewski, Laurent Gourvès, Jérôme Lang, Julien Lesca, **JM** : How Hard Is It for a Party to Nominate an Election Winner ? IJCAI 2016.

# Voting under incomplete preferences

- for each voter : $P_i$ is a partial order on the set of candidates.
- $P = \langle P_1, \ldots, P_n \rangle$ incomplete profile
- completion of $P$ : voting profile

  $$T = \langle T_1, \ldots, T_n \rangle$$

  where each $T_i$ is a linear order extending $P_i$.
- $F$ voting rule (resolute or irresolute)

- $c$ is a possible winner if there exists a completion of $P$ for which $c$ is a winner.
- $c$ is a necessary winner if $c$ is a winner in every completion of $P$.

# Possible winners : missing candidates

Missing candidates The voters have expressed their votes on a set of candidates, and then some new candidates come in.

- Doodle : agents vote on a first set of dates, and then new dates become possible
- Recruiting committee : a preliminary vote is done before the last applicants are interviewed

| voter 1 | voter 2 | ... | voter n |
|---|---|---|---|
| $c$ | $b$ | | $b$ |
| $\downarrow$ | $\downarrow$ | | $\downarrow$ |
| $a$ | $c$ | ... | $a$ |
| $\downarrow$ | $\downarrow$ | | $\downarrow$ |
| $b$ | $a$ | | $c$ |

$(d, e\,?)$

# Possible winners : missing candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win ?*

- 12 voters ; initial candidates : $X = \{a, b, c\}$ ; one new candidate $y$.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners ?

| | |
|---|---|
| $a$ | 5 |
| $b$ | 4 |
| $c$ | 3 |
| $y$ | |

initial scores (before $y$ is taken into account)

# Possible winners : missing candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win ?*

- 12 voters ; initial candidates : $X = \{a, b, c\}$ ; one new candidate $y$.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners ?

| | | |
|---|---|---|
| **a** | 5 | $\rightarrow$ **5** |
| $b$ | 4 | $\rightarrow 4$ |
| $c$ | 3 | $\rightarrow 3$ |
| $y$ | | $\rightarrow 0$ |

nobody votes for $y$

# Possible winners : missing candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win ?*

- 12 voters ; initial candidates : $X = \{a, b, c\}$ ; one new candidate $y$.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners ?

| | | |
|---|---|---|
| $a$ | 5 | $\rightarrow$ 3 |
| **b** | 4 | $\rightarrow$ **4** |
| $c$ | 3 | $\rightarrow$ 3 |
| $y$ | | $\rightarrow$ 2 |

2 who voted for $a$ now vote for $y$

# Possible winners : missing candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win ?*

- 12 voters ; initial candidates : $X = \{a, b, c\}$ ; one new candidate $y$.
- plurality with tie-breaking priority $a > b > c > y$
- Who are the possible winners ?

| | | | |
|---|---|---|---|
| $a$ | 5 | $\rightarrow 2$ | 3 who voted for $a$ |
| $b$ | 4 | $\rightarrow 2$ | and 2 who voted for $b$ |
| **c** | 3 | $\rightarrow 3$ | now vote for $y$, who wins ! |
| $y$ | | $\rightarrow \mathbf{5}$ | $c$ cannot win |

# Possible winners : missing candidates

- (For reasonable voting rules) all new candidates must be possible winners.
- *Who among the initial candidates can win ?*

- 12 voters ; initial candidates : $X = \{a, b, c\}$ ; two new candidates $y_1, y_2$
- plurality with tie-breaking priority $a > b > c > y_1 > y_2$
- Who are the possible winners ?

| | | | |
|---|---|---|---|
| $a$ | 5 | 2 | |
| $b$ | 4 | 2 | |
| $c$ | 3 | 3 | $c$ wins |
| $y$ | | 3 | |
| $y'$ | | 2 | |

# Possible winners : missing candidates

| | | |
|---|---|---|
| $a$ | 5 | $\rightarrow$ 2 |
| $b$ | 4 | $\rightarrow$ 2 |
| **c** | 3 | $\rightarrow$ 3 |
| $y$ | | $\rightarrow$ **5** |

| | | |
|---|---|---|
| $a$ | 5 | 2 |
| $b$ | 4 | 2 |
| $c$ | 3 | 3 |
| $y$ | | 3 |
| $y'$ | | 2 |

General result for plurality :

- $P_X$ initial profile on set of initial candidates $X$
- $ntop(P_X, x)$ number of voters who rank $x$ in top position in $P_X$ (plurality score of $x$ in $P_X$).

Then $x \in X$ is a possible winner for $P_X$ with respect to the addition of $k$ new candidates if and only if

$$ntop(P_X, x) \geq \frac{1}{k} . \sum_{x_i \in X} \max(0, ntop(P_X, x_i) - ntop(P_X, x))$$

- characterization and computation of possible winners for many voting rules : Chevaleyre, Lang, Maudet and Monnot (2010) ; Xia, Lang and Monnot (2011) ; Chevaleyre, Lang, Maudet, Monnot and Xia (2012)

# Voting under incomplete preferences : other works

- compilation-communication protocols (Chevaleyre, Lang, Maudet, Monnot 11) : how can we compile the information about the preferences of voters over the initial candidates, and depending in this compilation, what do we have to elicit about the new candidates ?
- possible and necessary winners in approval voting (Barrot, Gourvès, Lang, Monnot, Ries 13) : given a profile $(\succ_1, \ldots, \succ_n)$ of rankings and assuming voters cast approval votes that are consistent with their preferences, who are the possible approval winners ? what are the possible sets of approval co-winners ?
- voting with primaries (Faliszewski, Gourvès, Lang, Lesca, Monnot) : candidates are split between parties, each party nominates exactly one candidate for the final election : how hard is it to decide if (1) there is a set of nominees such that a candidate from a party $p$ wins in the final election ? (2) if a candidate from $p$ always wins, irrespective who is nominated ?

# Resource allocation and fairness

- Laurent Gourvès, **JM**, Lydia Tlilane : Approximate Tradeoffs on Matroids. ECAI 2012.
- Laurent Gourvès, **JM**, Lydia Tlilane : A Matroid Approach to the Worst Case Allocation of Indivisible Goods. IJCAI 2013.
  \+ journal version in TCS, 2015.
- Bruno Escoffier, Laurent Gourvès, **JM** : Fair solutions for some multiagent optimization problems. Auton. Agents Multi Agent Syst. 2013
- Laurent Gourvès, **JM**, Lydia Tlilane : Near Fairness in Matroids. ECAI 2014.
- Diodato Ferraioli, Laurent Gourvès, **JM** : On regular and approximately fair allocations of indivisible goods. AAMAS 2014.
- Haris Aziz, Péter Biró, Jérôme Lang, Julien Lesca, **JM** : Optimal Reallocation under Additive and Ordinal Preferences. AAMAS 2016
  \+ journal version in TCS, 2019.
- Laurent Gourvès, **JM** : Approximate Maximin Share Allocations in Matroids. CIAC 2017.
  \+ journal version in TCS, 2019.

# Resource allocation and fairness

- agents $N = \{1, \ldots, n\}$
- indivisible goods $O = \{o_1, \ldots, o_m\}$
- normalized additive utilities : $u_i(o_j) \in \mathbb{R}^+$ with $\sum_j u_i(o_j) = 1$
- for $A \subseteq O$, $u_i(A) = \sum_{o_j \in A} u_i(o_j)$
- allocation $\pi : N \to 2^O$ with $\pi(i) \cap \pi(j) = \emptyset$ for $i \neq j$
- maxmin allocation : $\pi$ maximizing $\min_i u_i(\pi(i))$

If $\alpha = \max_i \max_j u_i(o_j)$ is the maximal valuation assigned by an agnt to a single good then what is a lower bound $W_n(\alpha)$ on $\min_i u_i(\pi(i))$ ?

- $W_n(1) = 0$
- $W_n(1/n) = 1/n$
- inbetween ?

(Gourvès, Monnot and Tlinane 2013/2015) :

- improve previously known bounds
- polynomial algorithm giving each agent $i$ at least $W_n(\alpha_i)$
- generalization beyond resource allocation, to matroid-based domains

# Resource allocation and fairness

- for each individual $i$, the *maximin fair share* value of $i$ is the value she gives to the worst share of the best possible partition

$$MaxMinFS(i) := \max_{\pi} \min_{j} u_i(\pi(j))$$

| | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| *Ann* | 10 | 5 | 7 | 0 |
| *Bob* | 9 | 6 | 7 | 2 |

- $MaxMinFS(Ann) = 10$
- $MaxMinFS(Bob) = 11$

| | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| *Ann* | 10 | 5 | 7 | 0 |
| *Bob* | 9 | 6 | 7 | 2 |

- $\pi$ satisfies the *maxmin fair share* property if each individual obtains at least her maxmin fair share value.
- computing the maximin fair share if an agent is hard
- (Gourvès and Monnot, 2017/19) : polynomial approximations + generalization to matroids

# Resource allocation and fairness

- (Escoffier, Gourvès and Monnot, 2013) : maxmin collective combinatorial optimisation problems, especially spanning trees for collective network design.
- (Ferraioli, Gourvès and Monnot, 2014) : finding a maxmin allocation under the condition that each agent receives the same number of goods (regularity).
- (Aziz, Biró, Lang, Lesca, Monnot, 2016/19) : Pareto-efficient reallocation under additive/responsive preferences
  - finding an arbitrary Pareto-optimal allocation is easy but checking whether an allocation is Pareto-optimal can be hard
  - equivalent to checking that the allocated objects cannot be *reallocated* in such a way that one agent prefers her new allocation to the old one and no agent prefers the old one to tne new one.
  - additive utilities : hardness results and polynomial-time algorithms under different restrictions
  - responsive preferences : characterizations + polynomial algorithm

# Multiwinner voting and proportional representation

- Ioannis Caragiannis, Laurent Gourvès, **JM** : Achieving Proportional Representation in Conference Programs. IJCAI 2016.
- Dimitris Fotakis, Laurent Gourvès, **JM** : Conference Program Design with Single-Peaked and Single-Crossing Preferences. WINE 2016.
- Haris Aziz, Jérôme Lang, **JM** : Computing Pareto Optimal Committees. IJCAI 2016 : 60-66
- Jérôme Lang, **JM**, Arkadii Slinko, William S. Zwicker : Beyond Electing and Ranking : Collective Dominating Chains, Dominating Subsets and Dichotomies. AAMAS 2017.
- Haris Aziz, **JM** : Computing and testing Pareto optimal committees. Auton. Agents Multi Agent Syst. (2020)

# Proportional Conference Program Design

Input :

- $N = \{1, \ldots, n\}$ agents (participants)
- $X = \{x_1, \ldots, x_m\}$ items (papers)
- $k \in \mathbb{N}^*$ (number of slots)
- $q \in \mathbb{N}^*$ (number of rooms) such that $m \geq kq$
- $u_i : X \to \mathbb{R}^+$ utility function of agent $i$

Output :

- $\mathcal{S}$ collection of $k$ disjoint subsets $S_1, \ldots, S_k$ of $X$ with $|S_j| = q$ for all $j$
- utility of agent $i$ for program $\mathcal{S}$ : $u_i(\mathcal{S}) = \sum_{j=1}^{k} \max_{x \in S_j} u_i(x)$
- find a solution maximizing social welfare : find $S_1, \ldots, S_k$ maximizing

$$\left( \sum_{i=1}^{n} u_i(\mathcal{S}) = \right) \sum_{i=1}^{n} \sum_{j=1}^{k} \max_{x \in S_j} u_i(x)$$

# Proportional Conference Program Design

Input :

- $N = \{1, \ldots, n\}$, $X = \{x_1, \ldots, x_m\}$
- $k \in \mathbb{N}^*$, $q \in \mathbb{N}^*$, $m \geq kq$
- $u_i : X \to \mathbb{R}^+$

Output : $\mathcal{S}$ collection of $k$ disjoint subsets $S_1, \ldots, S_k$ of $X$ with $|S_j| = q$ for all $j$, maximizing $\sum_{i=1}^{n} \sum_{j=1}^{k} \max_{x \in S_j} u_i(x)$

| $i$ | $u_i(a)$ | $u_i(b)$ | $u_i(c)$ | $u_i(d)$ | $u_i(e)$ | $u_i(f)$ | $u_i(g)$ |
|---|---|---|---|---|---|---|---|
| 1 | 4 | 3 | 5 | 1 | 2 | 0 | 4 |
| 2 | 1 | 4 | 3 | 9 | 6 | 2 | 1 |
| 3 | 6 | 1 | 2 | 0 | 0 | 4 | 6 |

$S_1 = \{a, d\}, S_2 = \{b, f\}, S_3 = \{c, g\}$ :

| $i$ | $u_i(S_1)$ | $u_i(S_2)$ | $u_i(S_3)$ |
|---|---|---|---|
| 1 | 4 | 3 | 5 |
| 2 | 9 | 4 | 3 |
| 3 | 5 | 3 | 6 |

$$\mapsto \sum_{i=1}^{3} u_i(\mathcal{S}) = 42$$

# Proportional Conference Program Design

Input :

- $N = \{1, \ldots, n\}$ agents (participants)
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- $k \in \mathbb{N}^*$ (number of slots)
- $q \in \mathbb{N}^*$ (number of rooms) such that $m \geq kq$
- $u_i : X \rightarrow \mathbb{R}^+$ utility function of agent $i$

Output : $\mathcal{S}$ collection of $k$ disjoint subsets $S_1, \ldots, S_k$ of $X$ with $|S_j| = q$ for all $j$, maximizing $\sum_{i=1}^{n} \sum_{j=1}^{k} \max_{x \in S_j} u_i(x)$

Particular case : $k = 1$

- output : $S$ with $|S| = q$ maximizing $\sum_{i=1}^{n} \max_{x \in S} u_i(x)$
- *Chamberlin-Courant* multiwinner voting rule

# Proportional Conference Program Design

Input :

- $N = \{1, \dots, n\}$, $X = \{x_1, \dots, x_m\}$
- $k \in \mathbb{N}^*$, $q \in \mathbb{N}^*$, $m \geq kq$
- $u_i : X \to \mathbb{R}^+$

- NP-hardness already known for $k = 1$
- NP-hard also for $k = 2$, $m = 2q$ and dichotomous utilities (Caragiannis, Gourvès, Monnot 16)
- approximation algorithms (Caragiannis, Gourvès, Monnot 16)
- conference program design under single-peaked or single-crossing preferences : tractability + strategyproof mechanisms (Fotakis, Gourvès, Monnot 16)

# Computing Pareto Optimal Committees

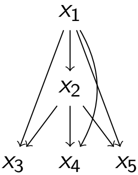
- $N = \{1, \ldots, n\}$ voters
- $X = \{x_1, \ldots, x_m\}$ candidates
- each voter expresses a weak order $\succsim_i$ over $X$ : $P = (\succsim_1, \ldots, \succsim_n)$.
- $S_k(X) = \{S \subset X : |S| = k\}$
- *preference extension* : $\succ_i^E$ extension of $\succsim_i$ over $S_k(X)$ (with $\succsim_i^E = \succsim_i$ for $k = 1$)
- Examples : let $A, B \in S_k(X)$ ;
  - *responsive extension* : $A \succsim^R B$ if there is an bijection $f : X \to X$ such that for all $x \in A$, $x \succsim f(x)$
  - *leximax extension* : $A \succsim^{leximax} B$ if the best element in $A$ is preferred to the best element in $B$, or if they are equally good but the second best element in $A$ is preferred to the second best element in $B$, etc.
  - two other extensions
- for each of these preference extensions, characterise and compute Pareto-optimal committees in $S_k(X)$

# Collective Dominating Chains, Dominating Subsets and Dichotomies

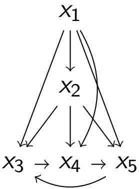
- Traditional voting setting : find one alternative (or a set of tied alternatives) based on the voters' preferences.
- Less traditional settings :
  1. electing a committee of $k$ persons (multiwinner election)
  2. finding a *ranked list* of $k$ candidates for an election based on party lists, or a ranked shortlist of $k$ names ;
  3. finding an optimal way of partitioning students between two or more groups with homogeneous level of ability in each group given their results on several tests.
  4. more complex settings : $k$ may not be fixed, the size of the partitions may be constrained etc.
- Define aggregation functions where the output can have any desired structure.
- Focus on some particular structures : *dominating chains*, *dominating subsets*, *dichotomies*.

# Collective Dominating Chains, Dominating Subsets and Dichotomies

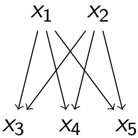
a *plain* dominating 2-chain



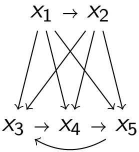

an *extended* dominating 2-chain




a plain dominating 2-subset




an extended dominating 2-subset




A *plain/extended dichotomy* is a plain/extended dominating $k$-subset for some $k \in \{1, \ldots, m-1\}$.